Damage and Healing in Fatigue Fracture

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We present an experimental and theoretical study of the fatigue failure of asphalt under cyclic compression. Varying the load amplitude, experiments reveal a finite fatigue limit below which the specimen does not break, while approaching the tensile strength of the material a rapid failure occurs. In the intermediate load range, the lifetime decreases with the load as a power law. We introduce two novel theoretical approaches, namely, a fiber bundle model and a fuse model, and show that both capture the major microscopic mechanisms of the fatigue failure of asphalt, providing an excellent agreement with the experimental findings. Model calculations show that the competition of damage accumulation and healing of microcracks gives rise to novel scaling laws for fatigue failure.

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The fracture of disordered media represents an important applied problem, with intriguing theoretical aspects. Statistical models have been successfully applied in the past to analyze fracture under quasistatic conditions, but the effect of cyclic loading is less explored [1]. Laboratory experiments reveal that fatigue failure under repeated loading is due to a combination of several mechanisms, among which damage growth, relaxation due to viscoelasticity, and healing of microcracks play an essential role [2, 3, 4]. Theoretical approaches have serious difficulties to capture all these mechanisms [2, 3, 4, 5, 6, 7] and fatigue life prediction is still very much an empirical science. Understanding this problem has crucial implications even for everyday applications. For example, fatigue failure occurring in roads due to repeated traffic loading cause main distress, limiting the lifetime of asphalt pavements.

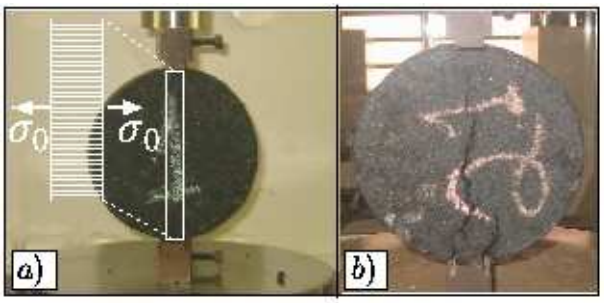

FIG. 1: Setup of the experiments. *a*) A cylindrical asphalt sample is subjected to diametrical compression applied periodically. FBM discretizes the region where tensile stress emerges (white rectangle) in terms of fibers. *b*) At complete failure a crack spans the cylinder along the load direction.

In this Letter we present a detailed experimental and theoretical study of the fatigue performance of hot mix asphalt (HMA). We carried out fatigue life tests of specimens measuring the accumulation of deformation with the number of loading cycles and the lifetime of specimens varying the load amplitude. To obtain a theoretical understanding of the experimental findings, we worked out two novel modelling approaches for fatigue failure, namely, a fiber bundle model [8, 9] and a fuse model [10]. We then show that both descriptions capture the stochastic nature of the fracture process, the immediate breaking of material elements and the cumulative effect of the loading history. Two physical mechanisms are considered which limit the accumulation of damage: a finite activation threshold of crack nucleation below which the local load does not contribute to the ageing of the material and healing of microcracks under compression, which leads to damage recovery. The analytical and numerical results of the model calculations provide a good quantitative agreement with the experimental findings. We show that the competition of nucleation and healing of microcracks leads to novel type of scaling laws for fatigue fracture with universal scaling exponents.

In order to obtain a quantitative characterization of the process of fatigue failure, we carried out fatigue life tests of asphalt under cyclic diametric compression of cylindrical specimens at a constant external load σ_0 (see Fig. 1). HMA is the primary material used to construct and maintain pavements and roadways due to its good mechanical performance and high durability. From the structural point of view asphalt is a combination of aggregates (usually crushed stone and sand), filler (cement, hydrated lime or stone dust) and a bituminous binder. Cylindrical samples of HMA were produced using the Marshall

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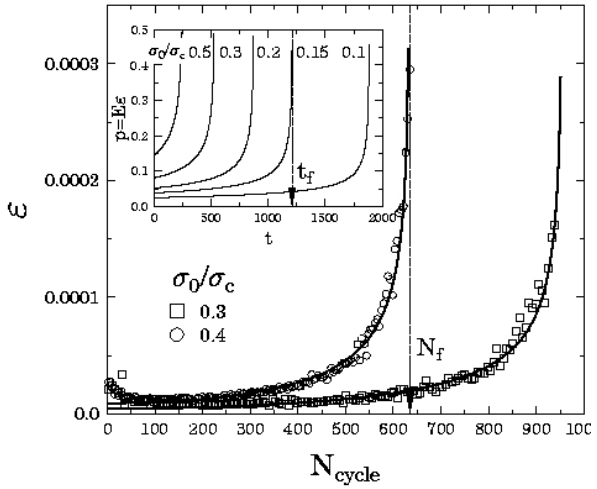

FIG. 2: Deformation as a function of the number of loading cycles. The continuous lines result from our theory. Inset: Load on the fibers in FBM as function of time at different values of σ_0/σ_c for uniformly distributed threshold values setting $\tau = \infty$ in Eq. (2).

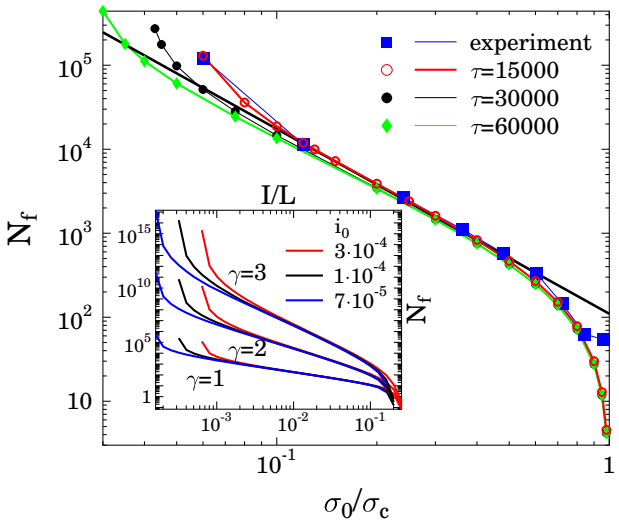

FIG. 3: *(Color online)* N_f as function of load σ_0/σ_c for FBM varying the value of τ. FBM provides an excellent fit of the experimental results with $\gamma = 2.0$, $\tau = 15000$, and $a = 0.01$. Fuse model results are presented in the inset varying the value of γ and the threshold current i_0.

method [11] and then loaded by a hydraulic device. Under repeated loading at a constant amplitude σ_0, the deformation ε was monitored as a function of the number of cycles N_{cycle}. Furthermore, the total number of cycles to complete failure N_f was measured varying σ_0. Figure 2 presents representative examples of $\varepsilon(N_{cycle})$ recorded at loads 30% and 40% of the tensile strength σ_c of the specimen. It can be observed that due to the gradual accumulation of damage, the deformation ε caused by the same load σ_0 monotonically increases until catastrophic failure occurs after a finite number of cycles N_f. The derivative of $\varepsilon(N_{cycle})$ also shows a monotonous increase and diverges when approaching the point of macroscopic failure. Increasing the external load the functional form of $\varepsilon(N_{cycle})$ remains the same, however, the lifetime of the specimen N_f gets shorter. The fatigue lifetime N_f measured at different fractions of the tensile strength σ_c (Fig. 3) reveals the existence of three distinct regimes. First, approaching the tensile strength of the material $\sigma_0/\sigma_c \to 1$ the lifetime N_f rapidly decreases indicating an immediate failure of the specimen. At the other extreme, a lower threshold value of the external load σ_l can be identified below which the specimen suffers only partial damage giving rise to an infinite lifetime (fatigue limit). In the intermediate regime the experimental results follow a power law known as Basquin law [2, 3, 4, 12]

$$N_f \sim \left(\frac{\sigma_0}{\sigma_c}\right)^{-\alpha}, \qquad (1)$$

where $\alpha = 2.2 \pm 0.1$, as shown in Fig. 3.

The experiments show that the fatigue crack growth is localized to a narrow region between the loading plates (see Fig. 1*b*) where locally a tensile stress emerges perpendicular to the external load [13]. To give a theoretical description of the failure process, we focus on this region and discretize it by a fiber bundle model (FBM) as illustrated in Fig. 1*a* [8, 9]. We consider a bundle of parallel linear elastic fibers with the same Young modulus E. Under diametrical compression of the disc-shaped specimen, the fibers experience a tensile loading and gradually fail due to immediate breaking or to the ageing of material elements [2]. More precisely, the following two mechanisms are considered: (I) fiber i $(i = 1, \ldots, N)$ breaks instantaneously at time t when its local load $p_i(t)$ exceeds the tensile strength p^i_{th} of the fiber. (II) All intact fibers undergo a damage accumulation process due to the load they have experienced. The amount of damage Δc_i occurred under the load $p_i(t)$ in a time interval Δt is assumed to have the form $\Delta c_i = a p_i(t)^\gamma \Delta t$ [13], hence, the total accumulated damage $c_i(t)$ until time t can be obtained by integrating over the entire loading history [13]. The exponent $\gamma > 0$ controls the damage accumulation rate and $a > 0$ is a scale parameter. The fibers can only tolerate a finite amount of damage and break when $c_i(t)$ exceeds a threshold value c^i_{th}. The two breaking thresholds p^i_{th} and c^i_{th} are random variables with a joint probability density function $h(p_{th}, c_{th})$. Assuming independence of the two breaking modes, h can be factorized into a product $h(p_{th}, c_{th}) = f(c_{th})g(p_{th})$, where $f(c_{th})$ and $g(p_{th})$ are the probability densities and $F(c_{th})$ and $G(p_{th})$ the cumulative distributions of the breaking thresholds p_{th} and c_{th}, respectively. For simplicity, after each breaking event the load of the broken fiber is equally redistributed over the intact ones irrespective of their distance from the failure point (global load sharing)

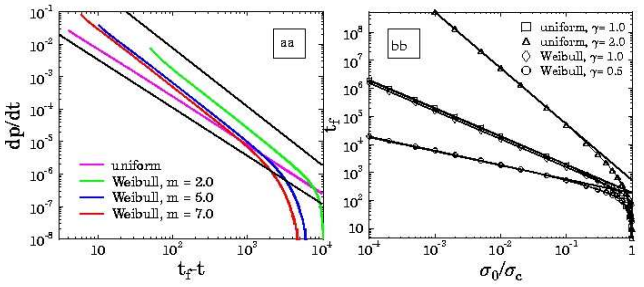

FIG. 4: *(Color online) a*) Deformation rate dp/dt as function of the time to macroscopic failure for different disorder distributions and $\gamma = 1$. Straight lines are drawn to guide the eye with slopes 1.5 and 1.8. *b*) t_f as a function of σ_0/σ_c for uniform and Weibull distributions ($m = 2.0$) varying the value of γ. The slope of the straight lines is equal to the value of the corresponding exponent γ.

[8, 9].

Under a constant tensile load σ_0, the load on a single fiber p_0 is initially determined by the quasi-static constitutive equation of FBM $\sigma_0 = [1 - G(p_0)]\, p_0$ [8, 9]. The external load σ_0 must fall below the tensile strength of the bundle $\sigma_0 < \sigma_c$, otherwise the bundle will fail immediately. As time elapses, the fibers accumulate damage and break due to their finite damage tolerance. These breakings, however, increase the load on the remaining intact fibers which in turn induce again immediate breakings. This way, in spite of the independence of the threshold values p_{th} and c_{th}, the two breaking modes are dynamically coupled, gradually driving the system to macroscopic failure in a finite time t_f at any load values σ_0. Healing of microcracks can be captured in the model by introducing a finite range τ for the memory, over which the loading history contributes to the accumulated damage [3, 14]. Finally, the evolution equation of the system can be cast in the form

$$\sigma_0 = [1 - F(a \int_0^t e^{-\frac{(t-t')}{\tau}} p(t')^\gamma dt')]\, [1 - G(p(t))]\, p(t), \tag{2}$$

where the integral in the argument of F provides the accumulated damaged at time t taking into account the finite range of memory by the exponential term [14]. In principle, the range of memory τ can take any positive value $\tau > 0$ such that during the time evolution of the bundle the damage accumulated during the time interval $t' < t - \tau$ heals. Equation (2) is an integral equation which has to be solved for the load $p(t)$ on the intact fibers at a given external load σ_0 with the initial condition $p(t = 0) = p_0$. The product in Eq. (2) arises due to the independence of the two breaking thresholds. We note that Eq. (2) recovers the usual constitutive behavior of FBM [8, 9] when damage accumulation is suppressed either by increasing the exponent γ or decreasing the range of memory $\tau \to 0$.

The inset of Fig. 2 presents examples of the solution $p(t)$ of Eq. (2) obtained for breaking thresholds uniformly distributed in the interval $[0, 1]$ at different ratios σ_0/σ_c setting $\tau \to \infty$. Since $p(t)$ is simply related to the macroscopic deformation ε of the bundle $p(t) = E\varepsilon(t)$, these results can directly be compared to the experimental findings. The agreement seen in Fig. 2 is obtained using Weibull distributions $P(x) = 1 - \exp[-(x/\lambda)^m]$ for the two breaking thresholds in Eq. (2) with the same Weibull exponent m and scale parameter λ. Our calculations also reveal that approaching macroscopic failure, the derivative of $p(t)$ has a power law divergence as a function of the time to failure $\frac{dp}{dt} \sim (t_f - t)^{-\beta}$. Fig. 4*a* shows that the exponent β solely depends on the type of disorder. Specifically, for breaking thresholds distributed over a finite and infinite range, we obtain the exponents $\beta = 1.5 \pm 0.02$ and $\beta = 1.8 \pm 0.06$, respectively, defining two different universality classes of the fatigue failure.

It is possible to recover the Basquin law Eq. (1) from Eq. (2), *i.e.* it can be shown analytically that for $\sigma_0/\sigma_c << 1$ and $\tau \to \infty$ the lifetime of the system has a power law dependence on the external load $t_f \sim \left(\frac{\sigma_0}{\sigma_c}\right)^{-\gamma}$, where γ is the damage accumulation exponent, independent on the type of disorder. Figure 4*b* shows that the numerical results are in excellent agreement with the above analytic prediction.

Due to Eq. (2), without healing ($\tau \to \infty$) the cumulative effect of the loading history gives rise to a macroscopic failure of the system at any load. However, our experiments revealed that damage recovery caused by healing of microcracks results in a finite fatigue limit σ_l, below which the sample does not break. Since healing takes place in the polymer binder, it can be controlled by changing the temperature [3]. In our FBM the healing of microcracks is captured by the finite range of memory τ. This is illustrated in Fig. 5 presenting the lifetime t_f for a fixed load varying the value of τ over a broad range. It can be seen that by decreasing τ, the lifetime of the system increases and, due to the competition between nucleation of new microcracks and healing of the existing ones, a finite critical value τ_c emerges below which the system only suffers a partial damage and has an infinite lifetime $t_f \to \infty$. Plotting the lifetime t_f as function of the distance from the critical point τ_c (inset of Fig. 5) we find a power law dependence of t_f

$$t_f \sim (\tau - \tau_c)^{-\delta}, \tag{3}$$

where $\delta = 0.5 \pm 0.01$ is an universal exponent, independent of the type of disorder and of the damage accumulation exponent γ. Consequently, at a given temperature where the system is characterized by a fixed value of τ, a finite fatigue limit σ_l emerges at which (Fig. 3) the lifetime diverges [3].

Besides healing, another important mechanism which

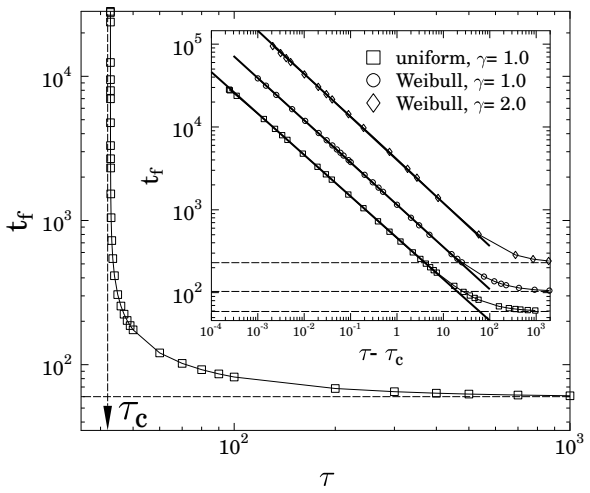

FIG. 5: Lifetime t_f of FBM at a constant load $\sigma_0/\sigma_c = 0.7$ varying τ. Approaching τ_c the lifetime diverges. Inset: t_f as a function of the distance from the critical range of memory $\tau - \tau_c$ for different disorder distributions and γ exponents. The horizontal lines indicate the value of $t_f(\tau = \infty)$.

limits damage accumulation is a finite activation threshold of microcrack nucleation. In order to study this effect we consider the random fuse model (RFM) [10] of fracture and extend it by introducing a history dependent ageing variable of fuses. We construct an $L \times L$ tilted square lattice of initially fully intact bonds with identical conductance but random failure thresholds i_c. The threshold values i_c are uniformly distributed between a small current value i_0 and 1 ($i_0 << 1$). For a given value of current I applied between two bus bars of the lattice, the local current through each bond is determined by solving numerically the Kirchhoff equations. Fuses burn out irreversibly when the current exceeds the local failure thresholds. This process is then followed by the recalculation of the current values. In order to capture fatigue cracking in the model, intact fuses are assumed to undergo an ageing process, modeled by a variable $A(t) = \sum_{t'=1}^{t} a(i(t') - i_0)^\gamma$. A fuse fails due to fatigue when $A(t) > A_{max}$, where A_{max} is a failure threshold uniformly distributed between $1 - b$ and $1 + b$ with $b = 0.1$. Comparing the two modelling approaches, the ageing variable $A(t)$ of RFM is analogous to the accumulated damage $c(t)$ of FBM, however, only current values above i_0 contribute to $A(t)$, which captures the finite activation threshold of microcrack nucleation. The inset of Fig. 3 demonstrates that RFM of ageing fuses provides qualitatively the same behavior as FBM, *i.e.* rapid failure at high current values I, a Basquin regime Eq. (1) at intermediate currents with an exponent equal to γ and a finite fatigue limit σ_l determined by the threshold current i_0.

In summary, our experiments on the fatigue failure of asphalt under cyclic compression revealed three regimes of the failure process depending on the load amplitude: instantaneous breaking, a Basquin regime of a power law decrease of lifetime and the existence of a fatigue limit below which no failure occurs. We introduced two novel modeling approaches both capturing the essential ingredients of the fatigue failure of bituminous materials. These models provide a comprehensive description of the experimental findings and additionally revealed novel scaling laws of fatigue fracture: approaching the macroscopic failure, the process of fatigue fracture accelerates and is characterized by a finite time power law singularity of the deformation rate. The exponent is different for bounded and unbounded disorder distributions defining two universality classes of fatigue fracture. Due to the interplay between damage and healing, at each load level a critical value of the range of memory emerges where the lifetime of the system has a universal power law divergence.

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